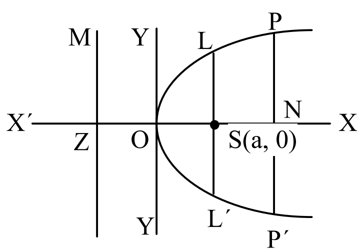
9-CONIC SECTION

Parabola :

The locus of a point which moves such that its distance from a fixed point is equal to its distance from a fixed straight line, i.e. e = 1 is called a parabola.



Its equation in standard form is $y^2 = 4ax$

(i) Focus S (a, 0)

(ii) Equation of directrix ZM is $x + a = 0$

(iii) Vertex is O (0, 0)

(iv) Axis of parabola is X′OX

Some definitions :

Focal distance : The distance of a point on parabola from focus is called focal distance. If $P(x_1, y_1)$ is on the parabola, then focal distance is $x_1 + a$.

Focal chord : The chord of parabola which passes through focus is called focal chord of parabola.

Latus rectum : The chord of parabola which passes through focus and perpendicular to axis of parabola is called latus rectum of parabola. Its length is 4a and end points are L(a, 2a) and L′(a, – 2a).

Double ordinate : Any chord which is perpendicular to the axis of the parabola is called its double ordinate.

- **Equation of tangent at** $P(x_1, y_1)$ is

  $yy_1 = 2a(x + x_1)$

  and equation of tangent in slope form is

  $$y = mx + \frac{a}{m}$$

  Here point of contact is $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

- **Equation of normal at** $P(x_1, y_1)$ is

  $$y - y_1 = \frac{-y_1}{2a}(x - x_1)$$

  and equation of normal in slope form is

  $y = mx - 2am - am^3$

  Here foot of normal is $(am^2, -2am)$

- The line $y = mx + c$ may be tangent to the parabola if $c = a/m$ and may be normal to the parabola if $c = -2am - am^3$.

- Chord of contact at point (x_1, y_1) is

  $yy_1 = 2a(x + x_1)$

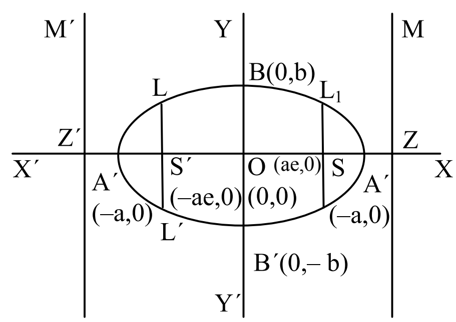
Ellipse :

If a point moves in a plane in such a way that ratio of its distances from a fixed point (focus) and a fixed straight line (directrix) is always less than 1, i.e. e < 1 called an **ellipse**

- Standard equation of an ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

  where $b^2 = a^2(1 - e^2)$

  Now, When a > b



In this position,

(i) Major axis 2a and minor axis 2b

(ii) Foci, S′(–ae, 0) and S(ae, 0) and centre O(0, 0)

(iii) Vertices A′ (–a, 0) and A(a, 0)

(iv) Equation of directries ZM and Z′M′ are

$$x \pm \frac{a}{e} = 0, Z\left(\frac{a}{e}, 0\right) \text{ and } Z'\left(-\frac{a}{e}, 0\right)$$

(v) Length of latus rectum is $\frac{2b^2}{a} = LL' = L_1L_1'$

- The coordinates of points of intersection of line $y = mx + c$ and the ellipse are given by

$$\left(\frac{-a^2m}{\sqrt{b^2 + a^2m^2}}, \frac{b^2}{\sqrt{b^2 + a^2m^2}}\right)$$

- Equation of tangents of ellipse in term of m is $y = mx \pm \sqrt{b^2 + a^2m^2}$

  and the line y = mx + c is a tangent of the ellipse, if $c = \pm\sqrt{b^2 + a^2m^2}$
- The length of chord cuts off by the ellipse from the line y = mx + c is

  $$\frac{2ab\sqrt{1+m^2}.\sqrt{a^2m^2+b^2-c^2}}{b^2+a^2m^2}$$
- The equation of tangent at any point (x_1, y_1) on the ellipse is

  $$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

  and at the point (a cos ϕ, b sin ϕ) on the ellipse, the tangents is

  $$\frac{x\cos\phi}{a} + \frac{y\sin\phi}{b} = 1$$

  Parametric equations of the ellipse are

  x = a cos θ and y = b sin θ.
- The equation of normal at any point (x_1, y_1) on the ellipse is

  $$\frac{(x-x_1)a^2}{x_1} = \frac{(y-y_1)b^2}{y_1}$$

  also at the point (a cos ϕ, b sin ϕ) on the ellipse, the equation of normal is

  $$ax \sec\phi - by \operatorname{cosec}\phi = a^2 - b^2$$
- Focal distance of a point $P(x_1, y_1)$ are $a \pm ex_1$
- Chord of contact at point (x_1, y_1) is

  $$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$
- Chord whose mid-point is (h, k) is

  $$\frac{hx}{a^2} + \frac{ky}{b^2} = \frac{h^2}{a^2} + \frac{k^2}{b^2} \quad \text{i.e. } T = S_1$$
- The locus of point of intersection of two perpendicular tangents drawn on the ellipse is $x^2 + y^2 = a^2 + b^2$. This locus is a circle whose centre is the centre of the ellipse and radius is length of line joining the vertices of major and minor axis. This circle is called "director circle".
- The eccentric angle of point P on the ellipse is made by the major axis with the line PO, where O is centre of the ellipse.
- (a) The sum of the focal distance of any point on an ellipse is equal to the major axis of the ellipse.

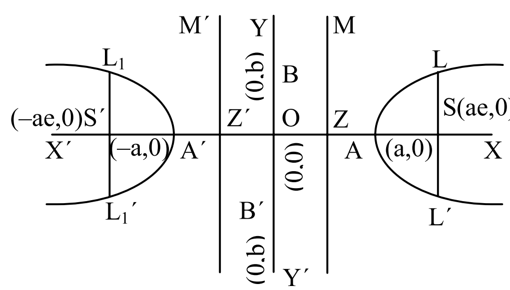
  (b) The point (x_1, y_1) lies outside, on or inside the ellipse f(x, y) = 0 according as $f(x_1, y_1) > = \text{or} < 0$.
- The locus of mid-point of parallel chords of an ellipse is called its **diameter** and its equation is $y = \frac{-b^2x}{a^2m}$ which is passes through centre of the ellipse.
- The two diameter of an ellipse each of which bisect the parallel chords of others are called **conjugate diameters.** Therefore, the two diameters $y = m_1x$ and $y = m_2x$ will be conjugate diameter if $m_1m_2 = -\frac{b^2}{a^2}$.

Hyperbola :

When the ratio (defined in parabola and ellipse) is greater than 1, i.e. e > 1, then the conic is said to be hyperbola.

Since the equation of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ differs from that of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in having $-b^2$, most of the results proved for the ellipse are true for the hyperbola, if we replace b^2 by $-b^2$ in their proofs. We therefore, give below the list of corresponding results applicable in case of hyperbola.

- Standard equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ where $b^2 = a^2(e^2 - 1)$



In this case,

- Foci are S (ae, 0) and S′(–ae, 0).
- Equation of directrices ZM and Z′M′ are

  $$x \mp \frac{a}{e} = 0, Z\left(\frac{a}{e}, 0\right) \text{ and } Z'\left(-\frac{a}{e}, 0\right)$$
- Transverse axis AA′ = 2a, conjugate axis BB′ = 2b.
- Centre O (0, 0).
- Length of latus rectum $LL' = L_1L_1' = \frac{2b^2}{a}$
- The difference of focal distance from any point $P(x_1, y_1)$ on hyperbola remains constant and is equal to the length of transverse axis. i.e.

  $S'P \sim SP = (ex_1 + a) - (ex_1 - a) = 2a$
- The equation of **rectangular hyperbola**

  $x^2 - y^2 = a^2 = b^2$ i.e. in standard form of hyperbola put a = b. Hence $e = \sqrt{2}$ for rectangular hyperbola.